

Light-cone QCD predictions for elastic ed -scattering in the intermediate energy region

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Abstract

The contributions of helicity-flip matrix elements to the deuteron form factors are discussed in the light-cone frame. Normalized $A(Q^2)$, $B(Q^2)$, $G_Q(Q^2)$ and T_{20} are obtained in a simple QCD-inspired model. We find that G_{+-}^+ plays an important role in $G_Q(Q^2)$. Our numerical results are consistent with the data in the intermediate energy region.

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I. INTRODUCTION

The electromagnetic properties of the deuteron have received extensive attention since it can be used to explore the quark and gluon degrees of freedom in the simplest nuclei. Many approaches are suggested to explain its electromagnetic form factors. Among them, the conventional meson-nucleon picture [1,2] gives a satisfying explanation in the low energy region, i.e. $Q^2 < 1 \text{ GeV}^2$. However, as mentioned by Arnold *et al.* [3], the experimental results are in sharp disagreement with the meson exchange calculations at higher momentum transfer (see Fig. 1). It means that quark and gluon degrees of freedom must be taken in account. Moreover, the success of dimensional counting rules [4] implies that perturbative QCD(pQCD) may give correct predictions at large momentum transfers. PQCD predicts [5] that, in the Light-cone frame(LCF), the helicity-zero to zero matrix element G_{00}^+ will be the dominant helicity amplitude at large Q^2 for elastic ed -scattering. Carlson and Gross [6] have pointed out that LCF helicity-flip amplitudes, G_{+0}^+ and G_{+-}^+ , are suppressed by factors of Λ_{QCD}/Q and $\Lambda_{\text{QCD}}^2/Q^2$, respectively. It is argued [7] that the dominance of G_{00}^+ , thus the validity of perturbative QCD predictions, begins at $Q^2 \sim 0.8 \text{ GeV}^2$. Assuming the helicity-zero to zero dominance, A simple ratio of form factors for the deuteron is predicted

$$G_C : G_M : G_Q = (1 - \frac{2}{3}\eta) : 2 : -1, \quad (1)$$

with the kinematic factor $\eta = Q^2/4M^2$.

Unfortunately, only to the lowest order, over 300 000 diagrams containing six fermion lines connected by five gluons are required to obtain the full elastic ed -scattering amplitude in pure pQCD approach. Such a calculation can be found in the work of Farrar, Huleihel, and Zhang [8], which shows that the direct application of pQCD to deuteron form factor at experimentally accessible momentum transfers has still a long way to go.

To make detailed predictions for deuteron electromagnetic form factors, we have suggested a model [9] in the intermediate energy region. Based on the reduced nuclear amplitude defined by Brodsky and Hiller [10], the model makes the points that a simple nuclear wave function can represent the data of the deuteron electromagnetic structure function $A(Q^2)$ and shows a scenario where G_{00}^+ is already dominant at Q^2 of 1 GeV^2 . Normalized G_{00}^+ can be extracted from fitting the data of $A(Q^2)$. It is pointed out [11] that the helicity-flip matrix element G_{+0}^+ can not be neglected in the expression of $B(Q^2)$. In addition to that, we find that G_{+-}^+ plays an important role in $G_Q(Q^2)$. Neglecting it will result in a contradiction with the data and the conventional meson-nucleon prediction in the low energy region. Due to the kinematical reason, the ratio of form factors in Eq. (1) should be modified even at extremely high Q^2 . We will model the helicity-flip matrix elements G_{+0}^+ and G_{+-}^+ in the intermediate energy region according to the pQCD predictions. The normalized structure functions $A(Q^2)$ and $B(Q^2)$, tensor polarization T_{20} , and form factors $G_C(Q^2)$ and $G_Q(Q^2)$ will be discussed on the basis of the normalized G_{00}^+ and the above model.

A QCD-inspired model for G_{00}^+ will be reviewed in Sec. II. Under a phenomenological consideration, the model is applied to obtain normalized $A(Q^2)$, $B(Q^2)$ and T_{20} in Sec. III, where G_Q is also discussed. The numerical results and summary are presented in Sec. IV and Sec. V, respectively.

II. A QCD-INSPIRED MODEL

For electron scattering on a deuteron, the Rosenbluth cross section is

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[A(Q^2) + B(Q^2) \tan^2\left(\frac{\theta}{2}\right) \right], \quad (2)$$

where $A(Q^2)$ and $B(Q^2)$ are determined by [12]:

$$A(Q^2) = G_C^2 + \frac{2}{3}\eta G_M^2 + \frac{8}{9}\eta^2 G_Q^2, \quad (3)$$

$$B(Q^2) = \frac{4}{3}\eta(1 + \eta)G_M^2, \quad (4)$$

with $\eta = Q^2/4M^2$, where M is the deuteron mass. The deuteron form factor $F_d(Q^2)$ is defined as $F_d(Q^2) \equiv \sqrt{A(Q^2)}$.

In the standard LCF, defined by [13] $q^+ = 0$, $q_y = 0$, and $q_x = Q$, the above form factors can be obtained from the plus component of three helicity matrix elements:

$$G_C = \frac{1}{2p^+(2\eta + 1)} \left[\left(1 - \frac{2}{3}\eta\right)G_{00}^+ + \frac{8}{3}\sqrt{2\eta}G_{+0}^+ + \frac{2}{3}(2\eta - 1)G_{+-}^+ \right], \quad (5a)$$

$$G_M = \frac{1}{2p^+(2\eta + 1)} \left[2G_{00}^+ + \frac{2(2\eta - 1)}{\sqrt{2\eta}}G_{+0}^+ - 2G_{+-}^+ \right], \quad (5b)$$

$$G_Q = \frac{1}{2p^+(2\eta + 1)} \left[-G_{00}^+ + \sqrt{\frac{2}{\eta}}G_{+0}^+ - \frac{\eta + 1}{\eta}G_{+-}^+ \right]. \quad (5c)$$

To avoid the complicated pQCD calculation, a model for the deuteron form factor $F_d(Q^2)$ has been suggested [9]. The point is that the hard kernel at large Q^2 is assumed to be the perturbative amplitude for the six quarks to scatter from collinear to the initial two-nucleon configuration to collinear to the final two-nucleon configuration, where each nucleon has roughly equal momentum. Since the gluon is a color octet, the single-gluon exchange between two color-singlet nucleon is forbidden. In the lowest order, the quark-interchange is necessary in addition to the one-gluon exchange between two nucleons. For the binding energy of the deuteron is small, we can divide roughly the kernel into two parts. One represents the interchange of quarks and the gluon exchange between two nucleons, which transfer about half of the transverse momentum of the virtual photon from the struck nucleon to the spectator nucleon. Another part is the inner evolution of two nucleons. The first part leads to the reduced form factor of the deuteron and the latter leads to the form factors of two nucleons. A vector boson(color singlet) with an effective mass M_b is introduced to represent the quark-interchange and the one-gluon exchange effects.

Assuming the G_{00}^+ dominance in the structure function $A(Q^2)$, we get [9]

$$G_{00}^+(Q^2) = 2(2\eta + 1)F_N^2(Q^2/4) \int [dx][dy] \phi_d^\dagger(x, Q) t_H(x, y, Q) \phi_d(y, Q), \quad (6)$$

where t_H is the hard scattering amplitude and ϕ_d is the body distribution amplitude of the deuteron defined by

$$\phi_d(x, Q) = \int [d\mathbf{k}_\perp] \Psi_d^{\text{body}}(x_i, \mathbf{k}_{\perp i}). \quad (7)$$

The argument for the nucleon form factor, F_N , is $Q^2/4$ since, in the limit of zero binding energy, each nucleon must change its momentum from $P/2$ to $(P+q)/2$. Using Brodsky-Huang-Lepage prescription [14] from a harmonic oscillator wave function, $A' \exp[-\frac{1}{2}\alpha^2 r^2]$, in the rest frame, the body wave function can be written as

$$\Psi_d^{\text{body}}(y, \mathbf{l}_\perp) = A \exp \left[-\frac{1}{2\alpha^2} \frac{\mathbf{l}_\perp^2 + m_N^2}{4y(1-y)} \right], \quad (8)$$

where A is determined by the normalization of the wave function and α by fitting the data of $A(Q^2)$. A direct calculation of diagrams in Fig. 2 gives

$$t_H(x, y, Q) = \frac{4M^2 g_{\text{eff}}^2}{xyQ^2 + M_b^2 - (x-y)^2 M^2} \frac{1}{xQ^2 + (\frac{1}{4} - (1-x)^2 M^2)}, \quad (9)$$

where g_{eff} is an effective coupling constant. We have taken the nucleon mass to be half of the deuteron mass in Eq. (9).

III. THE ROLE OF HELICITY-FLIP MATRIX ELEMENTS

In the intermediate energy region, G_{00}^+ dominates the charge form factor G_C , but not G_M and G_Q , because the kinematic factor η is still small. While $\eta \ll \frac{1}{2}$, the G_{+0}^+ contributions to both G_M and G_Q are enhanced by a factor of $1/\sqrt{2\eta}$. The G_{+-}^+ contribution to G_Q is enhanced by $1/\eta$. Although G_{+0}^+ and G_{+-}^+ are suppressed for dynamical reason, they may contribute significantly to G_M and G_Q because of the kinematic enhancement. Since G_C dominate $A(Q^2)$ while η is small, the G_{00}^+ dominance works very well in determining $A(Q^2)$. As for $B(Q^2)$, the helicity-flip matrix element G_{+0}^+ must be taken into account [11]. In $G_Q(Q^2)$, both G_{+0}^+ and G_{+-}^+ may be important.

As is well known, pQCD predicts that G_{+0}^+ and G_{+-}^+ are suppressed by factors Λ_{QCD}/Q and $\Lambda_{\text{QCD}}^2/Q^2$, respectively. The QCD scale Λ_{QCD} is around 200 MeV. In order to explore the role of G_{+0}^+ and G_{+-}^+ , we assume the following relations phenomenologically :

$$G_{+0}^+ = \frac{f}{\sqrt{2\eta}} G_{00}^+, \quad (10a)$$

$$G_{+-}^+ = c \frac{f^2}{2\eta} G_{00}^+, \quad (10b)$$

where f and c are two parameters. These relations are expected to be reasonable as $Q^2 \gtrsim 2M\Lambda_{\text{QCD}} \sim 0.8 \text{ GeV}^2$, where $2M\Lambda_{\text{QCD}}$ is a scale that determines the G_{00}^+ dominance [7]. Ref. [11] reveals that G_{+0}^+ contributes significantly to $G_M(Q^2)$. The G_{+0}^+ contribution to $G_Q(Q^2)$ was also discussed there. However, as mentioned above, G_{+-}^+ should be taken into account in determining G_Q due to the same reason as G_{+0}^+ . Detailed analysis shows that it plays an important role in the intermediate energy region. By assuming relation (10) we have picked up the contributions of G_{+0}^+ and G_{+-}^+ which may be comparable with that of G_{00}^+ to G_M and G_Q in the intermediate energy region. Nonleading contributions to G_{+0}^+ that

may enter at the same order of G_{+-}^+ are neglected for kinematic reason. Inclusion of these terms will change the magnitude of f slightly.

Substituting Eq. (10) into Eq. (5c), G_Q becomes

$$G_Q = \frac{1}{2P^+(2\eta + 1)} \left[-1 + \frac{f}{\eta} - \frac{\eta + 1}{\eta} \frac{cf^2}{2\eta} \right] G_{00}^+. \quad (11)$$

If G_{+-}^+ is suppressed strongly, $G_Q(Q^2)$ will be negative while the pQCD begin to be valid. Since it is positive at the origin, there must be a node in the region of $Q^2 < 1 \text{ GeV}^2$. It is in sharp disagreement with the meson-nucleon picture (e.g. see Ref. [1]) and the experimental data [15]. Another constraint on the parameters can be obtained from the data of $G_M(Q^2)$, which reveals that $G_M(Q^2)$ changes its sign at $Q^2 = Q_0^2 \sim 2\text{GeV}^2$. It turns out

$$2\eta_0 + (2\eta_0 - 1)f - cf^2 = 0, \quad (12)$$

with $\eta_0 = Q_0^2/4M^2$. Combining Eq. (11) and Eq. (12), it is shown that G_Q will keep positive at any momentum transfers if $c > 0.43$. Different choices of c will produce very different G_Q , but have little effect on $A(Q^2)$ and $B(Q^2)$.

IV. NUMERICAL RESULTS

At first, we assume the G_{00}^+ dominance in $A(Q^2)$, from which we get Eq. (6). By fitting the data of $A(Q^2)$ (see Fig. 1) we obtain the parameters: $M_b = 0.5\text{GeV}$, $\alpha = 0.21\text{GeV}$ and $\alpha_{\text{eff}} = g_{\text{eff}}^2/4\pi = 0.15$.

Then, given c , we can get the expressions of G_C , G_M , and G_Q by substituting Eq. (10) into Eqs. (5). As a demonstration, we will show the $c = 1$ case for simplicity. In this case f is $2\eta_0$.

$$G_C = \frac{G_{00}^+}{2(2\eta + 1)} \left[\frac{16}{3}\eta_0 + \frac{8}{3}\eta_0^2 + 1 - \frac{2}{3}\eta - \frac{4}{3}\frac{\eta_0^2}{\eta} \right], \quad (13a)$$

$$G_M = \frac{G_{00}^+}{2(2\eta + 1)} \left[2(2\eta_0 + 1)(1 - \frac{\eta_0}{\eta}) \right], \quad (13b)$$

$$G_Q = \frac{G_{00}^+}{2(2\eta + 1)} \left[-\frac{\eta_0^2}{\eta^2}(2\eta + 1) - (1 - \frac{\eta_0}{\eta})^2 \right], \quad (13c)$$

with $\eta_0 = Q_0^2/4M^2$. The inclusion of helicity-flip matrix elements has little effect on $A(Q^2)$ (see Fig. 1), but it changes the α_{eff} to 0.11. The normalized structure function $B(Q^2)$ is given in Fig. 3, where we have chosen the parameter $Q_0^2 = 1.85 \text{ GeV}^2$ (i.e. $\eta_0 = 0.13$) from fitting the data. From Eq. (13c) it is easy seen that G_Q keeps positive for all momentum transfers for G_{00}^+ is negative. For different choices of c , the normalized G_Q and T_{20} are shown in Fig. 4 and Fig. 5, respectively. The corresponding magnitudes of f are shown there, too. At very large momentum transfers, say $\eta \gg \eta_0$, the ratio of form factors will be slightly modified to

$$G_C : G_M : G_Q = ((1 + \frac{8}{3}f + \frac{2}{3}cf^2) - \frac{2}{3}\eta) : 2(1 + f) : -1. \quad (14)$$

V. SUMMARY

We have discussed the electromagnetic form factors of the deuteron in a QCD inspired model. Detailed kinematic analysis in LCF reveals that, provided the validity of pQCD, the helicity-zero to zero matrix element G_{00}^+ dominates the gross structure function $A(Q^2)$ in both the large and intermediate energy region, which is used to extract the normalized G_{00}^+ from a simple model for $A(Q^2)$. Further analysis shows that G_{+0}^+ and G_{+-}^+ are also important to determine other form factors. In the present work, G_{+0}^+ and G_{+-}^+ are modeled according to some pQCD predictions at high Q^2 . Normalized $B(Q^2)$ is obtained, whose vanishing at $Q_0^2 = 1.85 \text{ GeV}^2$ is used to determine the extent that helicity-flip matrix elements are suppressed to. To which extent the G_{+-}^+ is suppressed will strongly effect the behavior of G_Q in the intermediate energy region. We find that, If G_{+-}^+ is suppressed strongly, G_Q will change its sign twice and one of its nodes lies in the $Q^2 < 1 \text{ GeV}^2$ region. It is contrary to the meson-nucleon picture and experimental data. If $c > 0.43$, G_Q will keep positive for all momentum transfers in our model. Different choices of c have little effect on $A(Q^2)$ and $B(Q^2)$. As an example, we demonstrate a simple case by choosing $c = 1$, but a larger c shows a better fit for G_Q and T_{20} . Since the momentum transfers of available data are not high enough to determine the value of c reliably, we just show a qualitative result. At very high Q^2 , the ratio of form factors, $G_C : G_M : G_Q$, will be slightly modified. The ratio of G_C and G_Q is the same as predicted by pQCD as $Q^2 \rightarrow \infty$ [6]. It is apparent that G_M is bigger than that predicted by Brodsky and Hiller [7]. The kinematic factor $\sqrt{2\eta} - 1/\sqrt{2\eta}$ leads to the contributions of G_{+0}^+ to G_M in both the large and intermediate energy regions.

By assuming relation (10) we have picked up the leading contributions of G_{+0}^+ and G_{+-}^+ which may be comparable with that of G_{00}^+ to G_M and G_Q in the intermediate energy region. There are corrections from the next to leading order contributions to G_{+0}^+ which have the form similar to G_{+-}^+ . For G_Q these corrections can be neglected because of the lack of kinematic enhancement as G_{+-}^+ has. For G_M they enter at the same order as G_{+-}^+ . Inclusion of these terms will diminish the magnitude of parameter f in the same way as increasing parameter c . Since G_{+-}^+ is not important in G_M itself (i.e. f is insensitive to c), our conclusions will not suffer from neglecting them. Precise f can not be determined with these terms unknown. It is shown in Fig. 4 and Fig. 5 that f is around 0.2.

Since we obtain the normalized G_{00}^+ by assuming its dominance in $A(Q^2)$ and relation (10) by the validity of pQCD, our conclusions, except the ratio, are valid only in the intermediate energy region. It is found that G_C has a node [15] at $Q^2 = 0.75 \text{ GeV}^2$, which is predicted by conventional meson-nucleon method, too. Although our model can not predict the accurate position, it still shows that the node lies at somewhere $Q^2 < 1 \text{ GeV}^2$. At very large Q^2 , the contributions of hidden-color states should be taken into account to get a normalized G_{00}^+ . To some extent, our model shows a smooth connection with pQCD predictions in the high energy region and with traditional nuclear physics conclusions in the low energy region. Thus it can be expected to unify the predictions for the deuteron form factors from the low energy to large energy region. The experiments at CEBAF will be crucial to build a more realistic model in the intermediate energy region.

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FIGURES

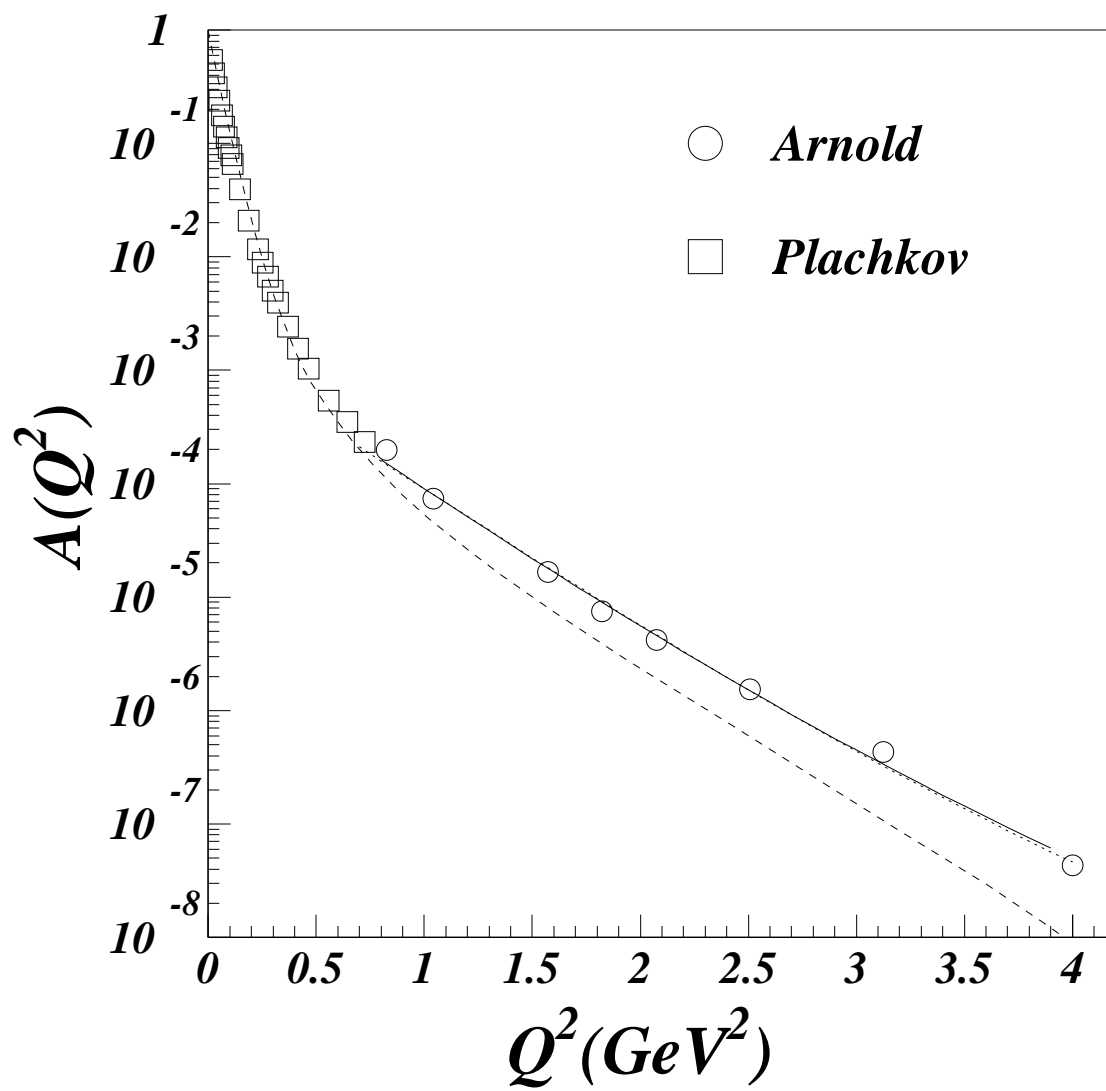
FIG. 1. Structure function $A(Q^2)$ of the elastic ed -scattering in our model(the solid line), with $M_b = 0.5$ GeV and $\alpha = 0.21$ GeV, The dashed line corresponds to the Paris potential calculation. Experimental data are taken from Ref. [3,16]. The dotted line is our result with the corrections from G_{+0}^+ and G_{+-}^+ , which is almost overlapping with the solid line.

FIG. 2. The hard scattering diagrams.

FIG. 3. Structure function $B(Q^2)$ (the solid line). The dashed line corresponds to the Paris potential calculation. Experimental data are taken from Ref. [17].

FIG. 4. The form factor G_Q . Experimental data are taken from Ref. [15].

FIG. 5. The tensor polarization T_{20} with scattering angle $\theta = 70^\circ$. The dashed line corresponds to the calculation with Paris potential.



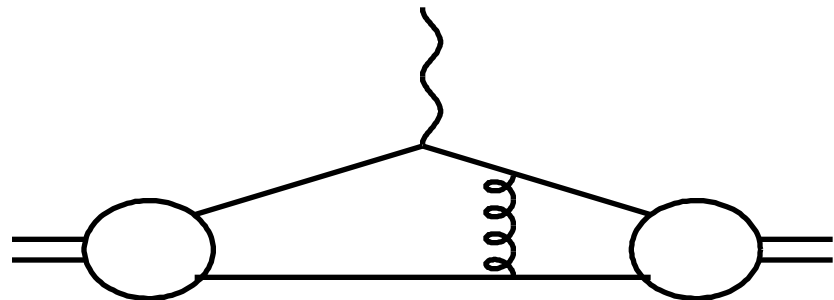
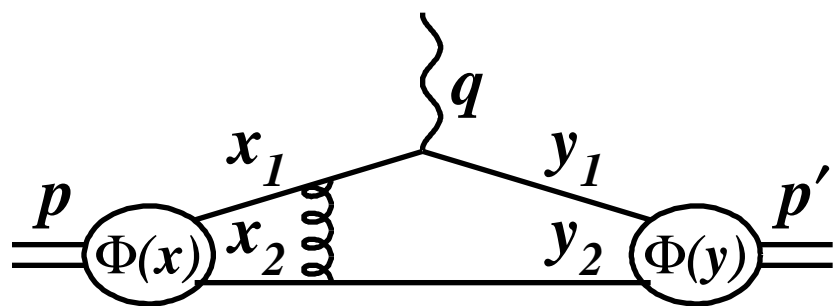


Fig.2

